

Elasticity and double integral

Given the following inverse demand function:

$$P = 10/Q + 1$$

And the following supply function:

$$P = 2Q$$

1. Find the price elasticity of demand at the equilibrium point. Classify the elasticity.
2. Formulate the double integral to calculate the producer surplus (the area below the equilibrium price and above the supply curve). Suggestion: draw a graph before setting up the integral.
3. Solve the double integral.

Solution

- Find the equilibrium by equating supply and demand:

$$10/Q + 1 = 2Q$$

$$Q = 2.5$$

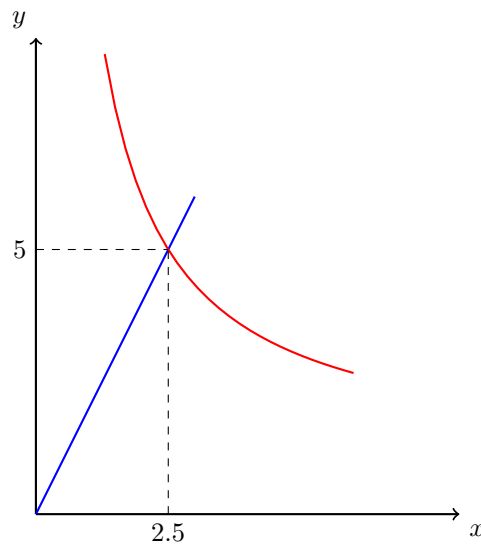
And the equilibrium price: $P = 5$. Calculate the price elasticity of demand, first express Q as a function of P :

$$Q = \frac{10}{P-1} = 10(P-1)^{-1}$$

$$\frac{\partial Q}{\partial P} \frac{P}{Q} = -10(P-1)^{-2} \frac{5}{2.5} = -10(5-1)^{-2} \frac{5}{2.5} = -1.25$$

It is elastic as $|-1.25| > 1$.

- Graphing:



We set up the following 2 possible double integrals:

$$\int_0^{2.5} \int_{2x}^5 dy dx = \int_0^5 \int_0^{y/2} dx dy$$

- Solving:

$$\int_0^{2.5} \int_{2x}^5 dy dx = \int_0^{2.5} 5 - 2x dx$$

Solving the integral:

$$5x - x^2$$

Evaluating at the boundaries:

$$5 \times 2.5 - (2.5)^2 = 6.25$$